# An Overview of Reduced Order Modelling Techniques at STS

**Onur Atak**

DI SW STS SDPRM MECH RTD XDT 3DROM



SIEMENS

# Reduced Order Modelling Overview

**Reduced Order Modelling or Model Order Reduction (MOR) are techniques to reduce the computational and/or storage requirements of simulation models while retaining the desired accuracy**

**The ROM Methods can be used in various contexts**

**1- Acceleration of classical 3D simulation workflows**

**2- Enabling System Level Modelling and Design Space Exploration**

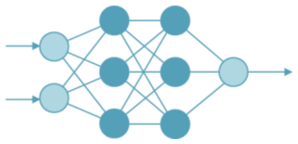
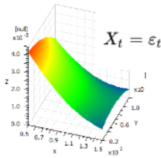
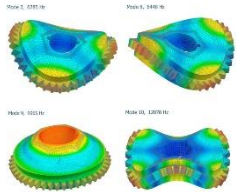
**3- Unlocking new applications such as Executable Digital Twins**

Based on the target application, the ROM method requirements differ: linear, nonlinear, static, dynamic, offline/online etc...

➢ **No single method to fit all!**

# Overview of ROM Methods

**Data Driven** → **Physics Based**

| Category | Machine learning | Mapping and interpolation models | Linear algebra | (Petrov)-Galerkin projections | Hyper-reduction and PMOR |
|---|---|---|---|---|---|
| **Technique** | **Neural networks** | **Response Surface Model Polynomial regression, FRF interpolation…** | **LTI models obtained via linearization** | **Modal CMS, Krylov subspace, multi-scale methods, POD ...** | **Hyper-reduction and PMOR** |
| **Examples** |  *multi-layer perceptrons, convolutional networks…* |  $X_t = \varepsilon_t + \sum_{i=1}^{p} \varphi_i X_{t-i} + \sum_{i=1}^{q} \theta_i \varepsilon_{t-i}$ *RSM, (auto)regressive models, Kriging, Loewner, Polymax* | $\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t)$ $\mathbf{y}(t) = \mathbf{C}\mathbf{x}(t) + \mathbf{D}\mathbf{u}(t)$ *low order linear models (transfer functions, state-space…)* |  *Mode sets, Krylov vectors, physics based ROMs* |  $\boldsymbol{K}(p) = \boldsymbol{K}_0 + \sum_i [\boldsymbol{K}_i^{\lambda} \lambda(E_i, \nu_i) + \boldsymbol{K}_i^{\mu} \mu(E_i, \nu_i)]$ *DEIM, ECSW, PGD, PMOR etc.* |
| **Data source** | 1D 3D CFD TEST | 1D 3D CFD TEST | 1D 3D CFD TEST | 1D 3D CFD TEST | 1D 3D CFD TEST |

*with* ***LTI****: Linear Time Invariant,* ***POD****: Proper Orthogonal Decomposition,* ***RSM****: Response Surface Model,* ***CMS****: Component Mode Synthesis*
***DEIM****: Discrete Empirical Interpolation Method,* ***ECSW****: Energy Conserving Sampling and Weighting,* ***PGD****: Proper Generalized Decomposition,* ***PMOR****: Parametric Model Oder Reduction*

# Reduced Order Modelling Methods

**The two main families of ROM methods are**

## Physics-Based Methods

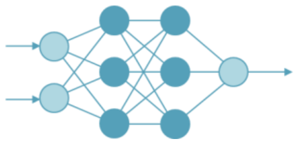
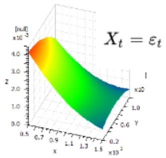
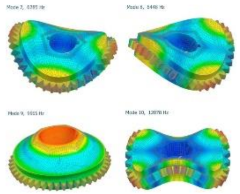
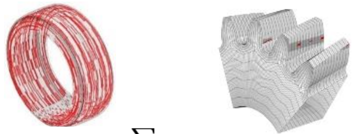
- Requires a priori knowledge of the system
- No training data required (linear systems) or limited training data required (nonlinear systems)
- Control over accuracy for extrapolation typically feasible/quite robust
- Can achieve modest to large simulation time speedups
- Can reduce a single iteration/frequency/time step of the simulation

## Data-driven Methods

- Does not require a priori knowledge of the system
- Requires large training data
- Control over accuracy for extrapolation might be difficult/less robust
- Can achieve extremely large simulation time speedups
- Typically used to reduce multiple iterations along frequency/time axis or parameters of simulations

MOR methods allow sharing of models in compact forms. They also provide IP Protection.

# Overview of ROM Methods

| **Data Driven** | | | | | **Physics Based** |
|---|---|---|---|---|---|
| **Category** | **Machine learning** | **Mapping and interpolation models** | **Linear algebra** | **(Petrov)-Galerkin projections** | **Hyper-reduction and PMOR** |
| **Technique** | **Neural networks** | **Response Surface Model Polynomial regression, FRF interpolation…** | **LTI models obtained via linearization** | **Modal CMS, Krylov subspace, multi-scale methods, POD ...** | **Hyper-reduction and PMOR** |
| **Examples** |  *multi-layer perceptrons, convolutional networks…* |  $X_t = \varepsilon_t + \sum_{i=1}^{p} \varphi_i X_{t-i} + \sum_{i=1}^{q} \theta_i \varepsilon_{t-i}$ *RSM, (auto)regressive models, Kriging, Loewner, Polymax* | $\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t)$ $\mathbf{y}(t) = \mathbf{C}\mathbf{x}(t) + \mathbf{D}\mathbf{u}(t)$ *low order linear models (transfer functions, state-space…)* |  *Mode sets, Krylov vectors, physics based ROMs* |  $\boldsymbol{K}(p) = \boldsymbol{K}_0 + \sum_i [\boldsymbol{K}_i^{\lambda} \lambda(E_i, \nu_i) + \boldsymbol{K}_i^{\mu} \mu(E_i, \nu_i)]$ *DEIM, ECSW, PGD, PMOR etc.* |
| **Data source** | 1D 3D CFD TEST | 1D 3D CFD TEST | 1D 3D CFD TEST | 1D 3D CFD TEST | 1D 3D CFD TEST |

*with* ***LTI****: Linear Time Invariant,* ***POD****: Proper Orthogonal Decomposition,* ***RSM****: Response Surface Model,* ***CMS****: Component Mode Synthesis*
***DEIM****: Discrete Empirical Interpolation Method,* ***ECSW****: Energy Conserving Sampling and Weighting,* ***PGD****: Proper Generalized Decomposition,* ***PMOR****: Parametric Model Oder Reduction*



SIEMENS

# (Petrov)-Galerkin Projection Methods

## MOR philosophy

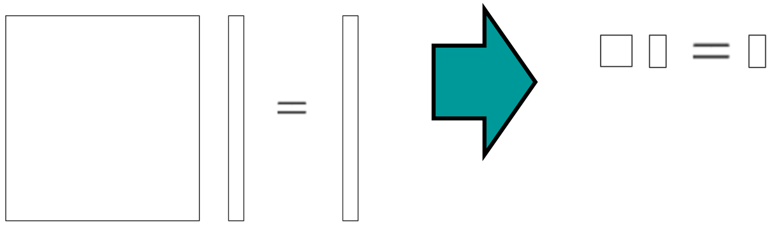
- Represent Full Order Model in reduced coordinates

FOM

$\mathbf{A}\mathbf{x} = \mathbf{b}$ $\quad \mathbf{A} \in \mathbb{R}^{n\times n}$

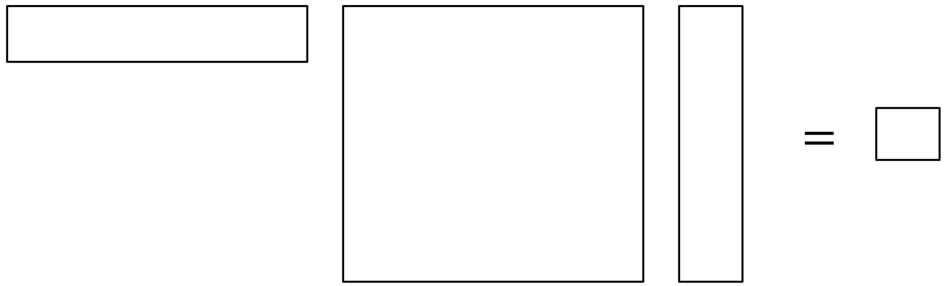
ROM

$\mathbf{A_r}\mathbf{x_r} = \mathbf{b_r}$ $\quad \mathbf{A_r} \in \mathbb{R}^{m\times m}$



- Challenge: find a proper projection basis **V**

$\mathbf{V^t}$ $\quad \mathbf{A}$ $\quad \mathbf{V}$ $\quad \mathbf{A_r}$



## Subspace Selection

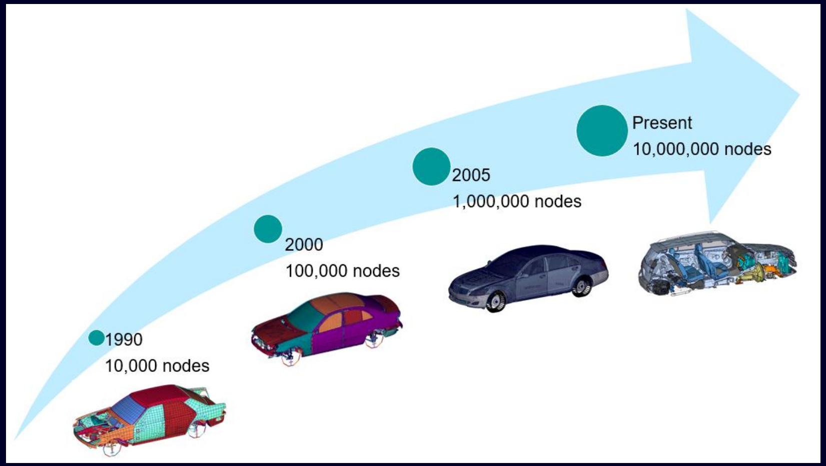
- Modal methods use the eigenvalues of the system matrices as basis. Enrichment with static modes are typically done (CMS)
  - The modes are physical
- Krylov Methods exploit combination of solution vectors (and their derivatives) in frequency domain
  - The subspace is “mathematical”, doesn’t have physical meaning typically
- POD methods accumulate snapshots in time-domain and orthogonalize them (apply SVD)



SIEMENS

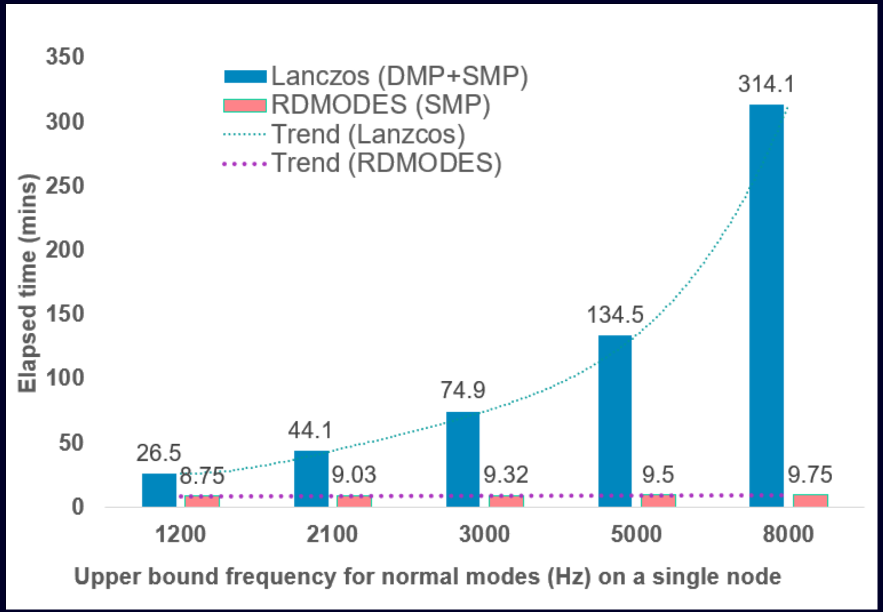
# RDMODES – Component Mode Synthesis (Recursive Domain Normal Modes)

## The Trend of Models:




## Divide and Conquer Strategy:

- A method where a model is automatically divided into finite number of components
- Modes for individual components are computed
- Model represented by component modes
- System modes synthesized



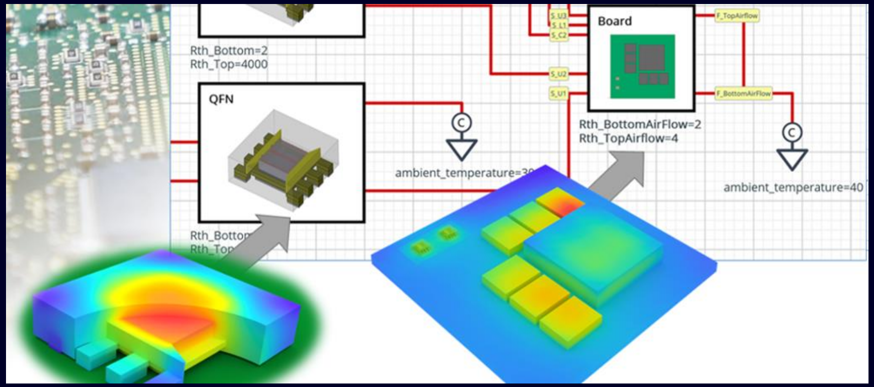

- Cost of Lanczos when frequency range is increased is almost quadratic
- Cost of eigenvalue extraction with RDMODES is linear with increase in frequency range of interest

*Available in Simcenter 3D Nastran -> Contact: Easwaran Viswanathan (DI SW STS SDPRM MECH STR DYN)*



SIEMENS

# Krylov Based Methods




## Method properties

- "Projection subspace" based on the solution vectors and its derivatives (Arnoldi process)
- Applicable to linear (or linearized) physics
- Available for 1st or 2nd order systems:
  - Structural Dynamics, Vibro-Acoustics, Thermal (conduction), Thermo-mechanical etc…
- High accuracy for a given input/output relation
- The efficiency depends on the number of uncorrelated sources

## Available Software

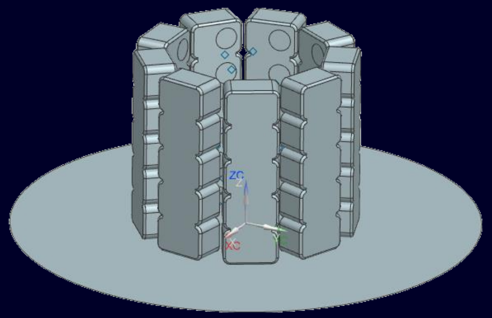
- Boundary Condition Independent ROM (BCI-ROM) of Simcenter Flotherm and Simcenter FloEFD uses Krylov method 1st order version
  - Valid for a range of Heat Transfer Coefficients
- Simcenter ROM Builder also supports Krylov methods
  - 1st order problems in December Release.
  - 2nd order systems support afterwards
  - Has an automatic adaptive snapshot selection algorithm

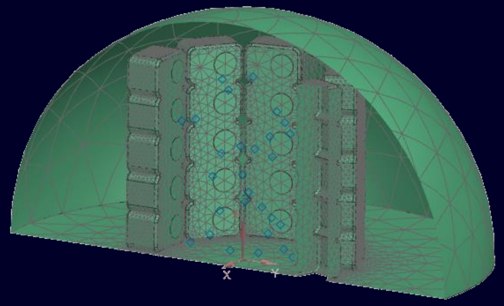


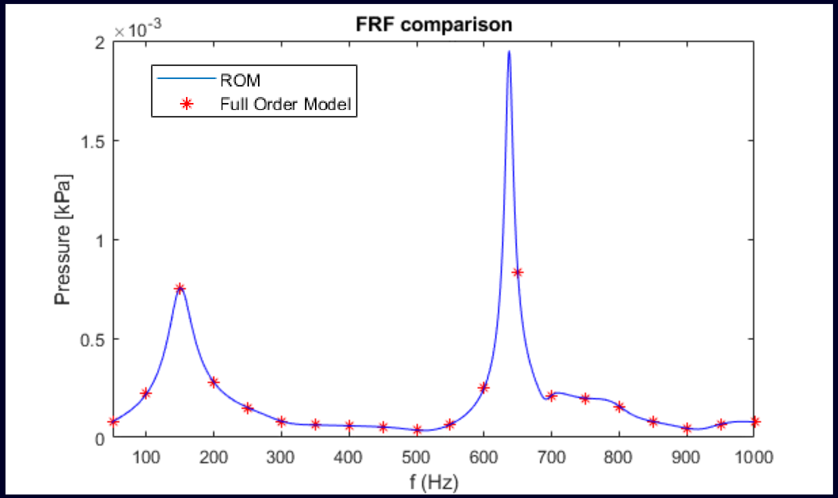
SIEMENS

# Application of SOAR to Unbounded (Vibro-)Acoustics

## Direct Field Acoustic Testing



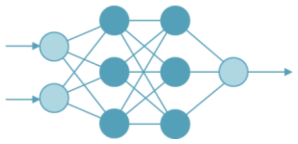
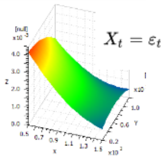
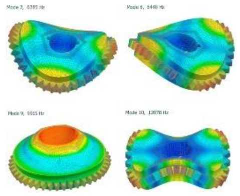





- Classical Modal approaches do not exist for the unbounded problems
- ~230k Dof FEMAO Model (equal to ~1 mil Dof linear FEM Model)
  - Includes Infinite Elements attached to surrounding hemisphere
  - reduced to 803 Dof (for 9 independent RHS excitations)
- The ROM is valid up to 1000 Hz with Error Tol $10^{-2}$
- 5 snapshots used (only 5 full system solves to create the ROM)
- ROM has constant impedance BC applied on the loudspeaker faces

*1- D. Bizzarri, O. Atak, S. van Ophem, H. Beriot, T. Tamarozzi, P. Jiranek, L. Scurria, Garcia de Miguel, M. Alvarez Blanco, K. Janssens, W. Desmet, "Model Order Reduction and Smart Virtual Sensing for Unbounded Vibro-Acoustics Using High Order FEM and Infinite Elements" Proceedings of ISMA 2022, Leuven, Belgium*

# Overview of ROM Methods

| Data Driven | | | | | Physics Based |
|---|---|---|---|---|---|
| **Category** | **Machine learning** | **Mapping and interpolation models** | **Linear algebra** | **(Petrov)-Galerkin projections** | **Hyper-reduction and PMOR** |
| **Technique** | **Neural networks** | **Response Surface Model Polynomial regression, FRF interpolation…** | **LTI models obtained via linearization** | **Modal CMS, Krylov subspace, multi-scale methods, POD ...** | **Hyper-reduction and PMOR** |
| **Examples** |  *multi-layer perceptrons, convolutional networks…* |  $X_t = \varepsilon_t + \sum_{i=1}^{p}\varphi_i X_{t-i} + \sum_{i=1}^{q}\theta_i \varepsilon_{t-i}$ *RSM, (auto)regressive models, Kriging, Loewner, Polymax* | $\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t)$ $\mathbf{y}(t) = \mathbf{C}\mathbf{x}(t) + \mathbf{D}\mathbf{u}(t)$ *low order linear models (transfer functions, state-space…)* |  *Mode sets, Krylov vectors, physics based ROMs* |   $\boldsymbol{K}(p) = \boldsymbol{K}_0 + \sum_i [\boldsymbol{K}_i^{\lambda}\lambda(E_i,\nu_i) + \boldsymbol{K}_i^{\mu}\mu(E_i,\nu_i)]$ *DEIM, ECSW, PGD, PMOR etc.* |
| **Data source** | 1D 3D CFD TEST | 1D 3D CFD TEST | 1D 3D CFD TEST | 1D 3D CFD TEST | 1D 3D CFD TEST |

*with **LTI**: Linear Time Invariant, **POD**: Proper Orthogonal Decomposition, **RSM**: Response Surface Model, **CMS**: Component Mode Synthesis*
***DEIM**: Discrete Empirical Interpolation Method, **ECSW**: Energy Conserving Sampling and Weighting, **PGD**: Proper Generalized Decomposition, **PMOR**: Parametric Model Oder Reduction*

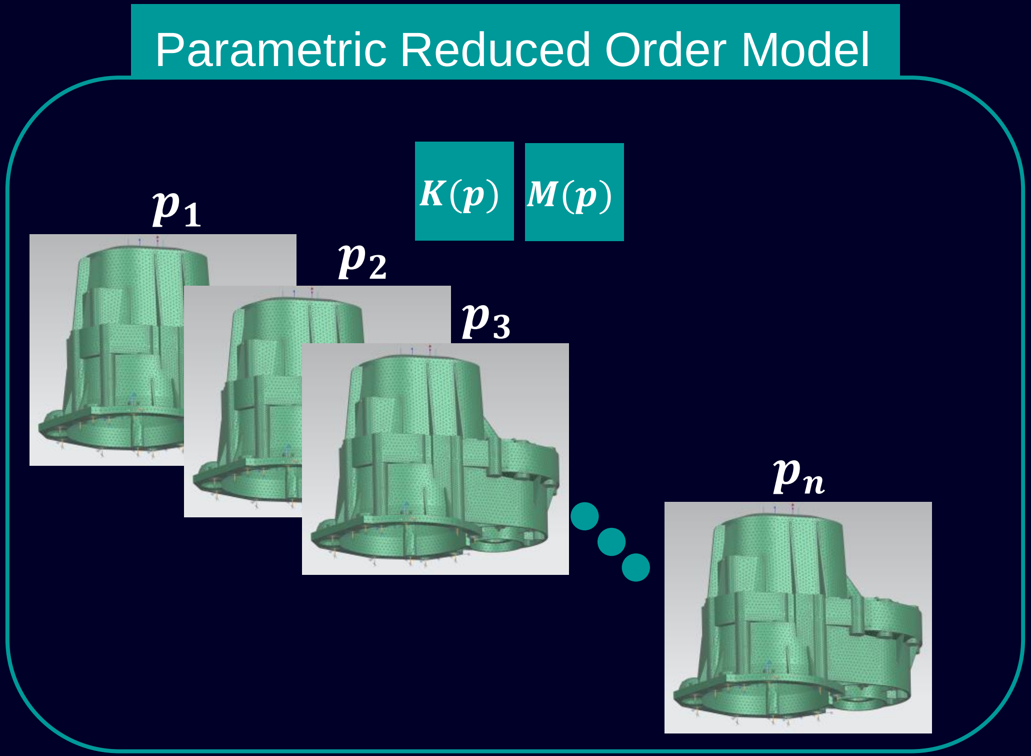
# Parametric Model Order Reduction (pMOR)

- Retains parameter dependencies on the ROM level
  - No need to call the solver after ROM creation

- Samples the parameter space and deduce affine dependencies

- Parameters that can be tackled:
  - Young's modulus and other constitutive coefficients
  - Density
  - Poisson Coefficient
  - Shell element thickness
  - Lumped mass, spring and damping values

- After constructing parametric reduced order model, any parameter value of interest can be directly evaluated






SIEMENS

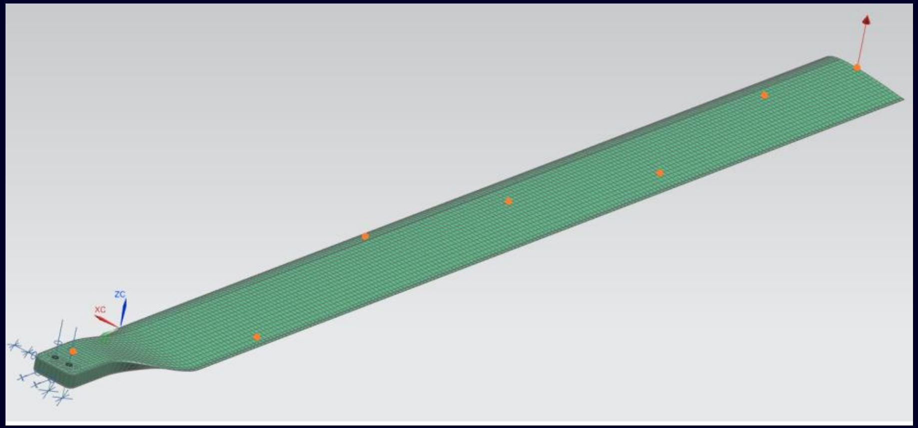
# Parametric Model Order Reduction (pMOR)

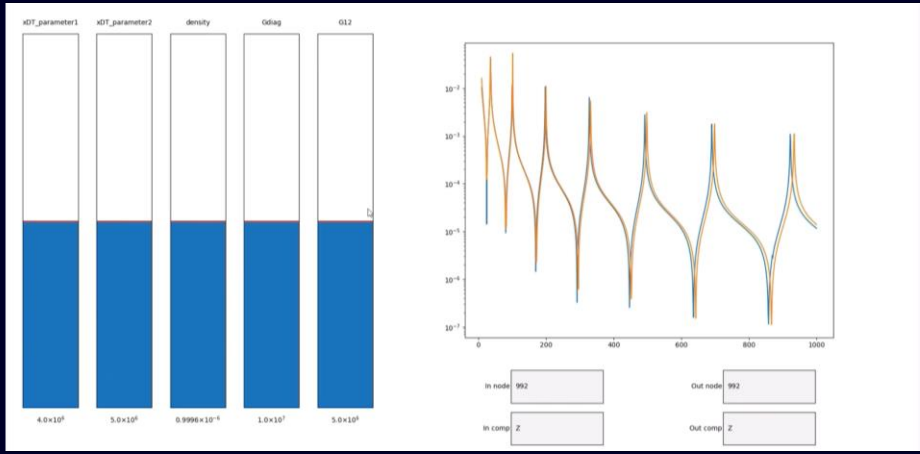
<u>Structural Dynamics Model of a Helicopter Blade</u>

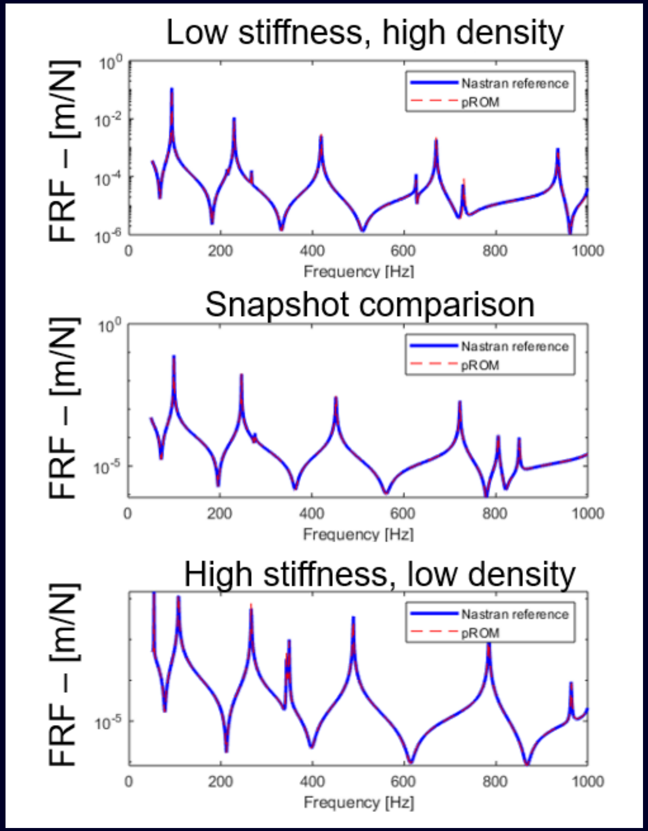
- Frequency [50:1:1000] Hz
- Structural damping G = 0,001
- Point force in Z-direction
- Output: X-Y-Z displacements in 7 nodes spread out over the blade

<u>Anisotropic material properties</u>

- 5 parameters:
  - G11-22, G12, G33 and rho
  - Connection stiffness: K1
- Parameters have a range of $\pm$ 30% around their reference values






1- Capalbo, Cristian Enrico, Daniel De Gregoriis, Tommaso Tamarozzi, Hendrik Devriendt, Frank Naets, Giuseppe Carbone, and Domenico Mundo. "Parameter, input and state estimation for linear structural dynamics using parametric model order reduction and augmented Kalman filtering." *Mechanical Systems and Signal Processing* 185 (2023): 109799.

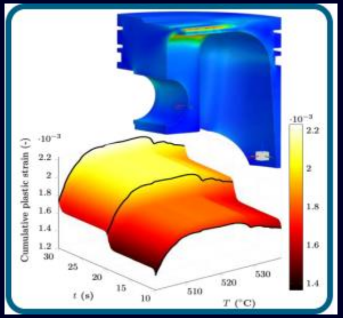
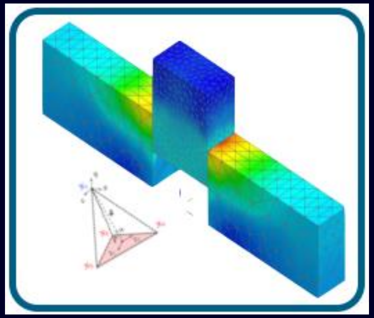
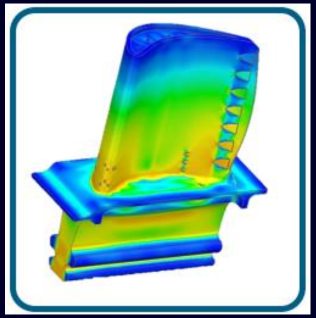


SIEMENS

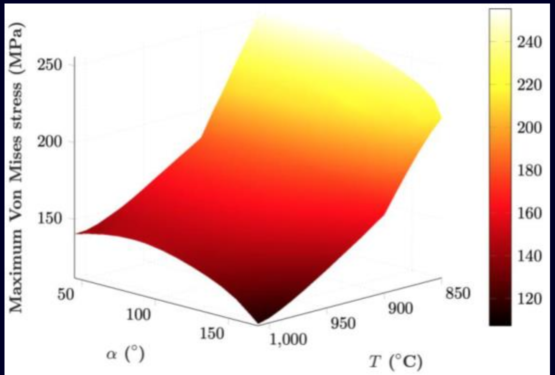
# LATIN-PGD for Quasi-static Nonlinear FE

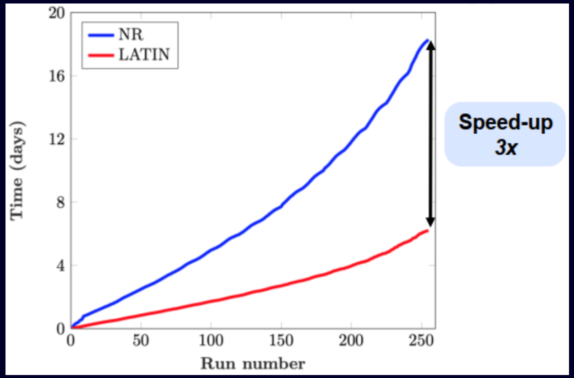
- Speed-up computations in a robust and weakly-intrusive manner with LATIN-PGD method
  - By using a new non-incremental solver : LATIN (replacing classical NR algorithm)
  - By adding Reduced-Order Modelling (ROM) technics : PGD

- Applicable to all non-linear behavioural laws (plasticity, creep,…)
  - all boundary non-linearities (contact)
  - all geometric non-linearities (large transformation)
  - all element types and boundary conditions etc…

Faster solution delivery for the multi-run computations: from 3 to 10 times faster



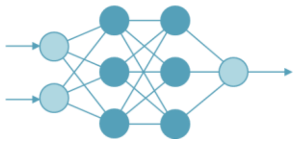
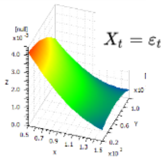
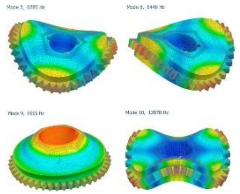




1- Scanff, Ronan, David Néron, Pierre Ladevèze, Philippe Barabinot, Frédéric Cugnon, and Jean-Pierre Delsemme. "Weakly-invasive LATIN-PGD for solving time-dependent non-linear parametrized problems in solid mechanics." *Computer Methods in Applied Mechanics and Engineering* 396 (2022): 114999.



SIEMENS

# Overview of ROM Methods

| Data Driven | | | | | Physics Based |
|---|---|---|---|---|---|
| **Category** | **Machine learning** | **Mapping and interpolation models** | **Linear algebra** | **(Petrov)-Galerkin projections** | **Hyper-reduction and PMOR** |
| **Technique** | **Neural networks** | **Response Surface Model Polynomial regression, FRF interpolation…** | **LTI models obtained via linearization** | **Modal CMS, Krylov subspace, multi-scale methods, POD ...** | **Hyper-reduction and PMOR** |
| **Examples** |  *multi-layer perceptrons, convolutional networks…* |  $X_t = \varepsilon_t + \sum_{i=1}^{p} \varphi_i X_{t-i} + \sum_{i=1}^{q} \theta_i \varepsilon_{t-}$ *RSM, (auto)regressive models, Kriging, Loewner, Polymax* | $\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t)$ $\mathbf{y}(t) = \mathbf{C}\mathbf{x}(t) + \mathbf{D}\mathbf{u}(t)$ *low order linear models (transfer functions, state-space…)* |  *Mode sets, Krylov vectors, physics based ROMs* |  $\boldsymbol{K}(p) = \boldsymbol{K}_0 + \sum_i [\boldsymbol{K}_i^{\lambda} \lambda(E_i, \nu_i) + \boldsymbol{K}_i^{\mu} \mu(E_i, \nu_i)]$ *DEIM, ECSW, PGD, PMOR etc.* |
| **Data source** | 1D 3D CFD TEST | 1D 3D CFD TEST | 1D 3D CFD TEST | 1D 3D CFD TEST | 1D 3D CFD TEST |

*with* ***LTI****: Linear Time Invariant,* ***POD****: Proper Orthogonal Decomposition,* ***RSM****: Response Surface Model,* ***CMS****: Component Mode Synthesis*
***DEIM****: Discrete Empirical Interpolation Method,* ***ECSW****: Energy Conserving Sampling and Weighting,* ***PGD****: Proper Generalized Decomposition,* ***PMOR****: Parametric Model Oder Reduction*



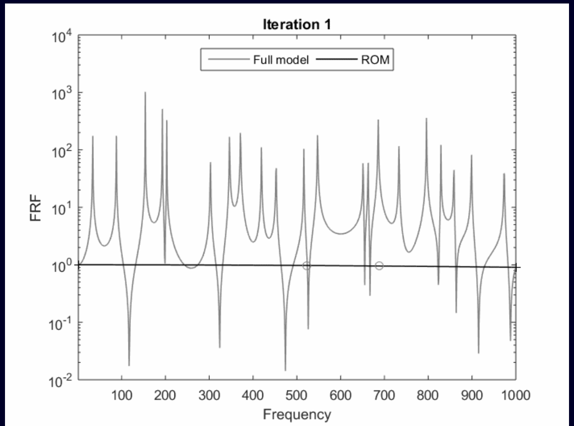
SIEMENS

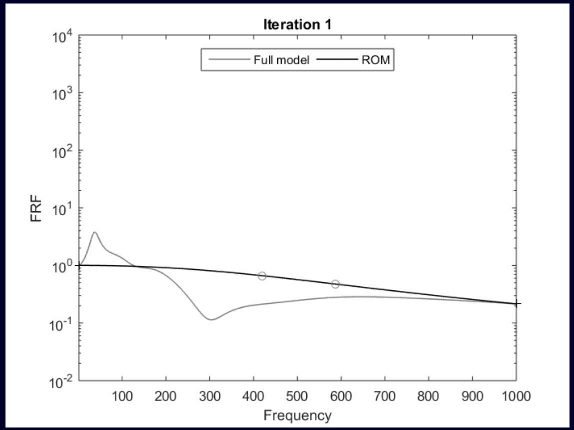
# A Fast Frequency Sweep Method via the Loewner Framework: MatrixFree Method

## What is it?

- **MatrixFree** is a non-intrusive method that **accelerates FRF calculations**
  - *For instance, instead of 1000 simulations to calculate 1-1000Hz, MatrixFree would only require 50 simulations)*
- It is a black-box method (no need for system matrices)
- It is iterative (adaptively enriched), it uses rational interpolation functions (called Loewner framework)
- Finds the “dynamics of the system” automatically
  - No need for user to define different frequency steps per octave band
- Can tackle any kind of frequency dependent property

## Numerical Example




| | Bare | CLD |
|---|---|---|
| # Full DOF | 6482 | 38601 |
| # Iterations | 27 | 11 |
| # Frequencies (red.) | 54 | 22 |
| # Frequencies (full) | 999 | 999 |
| **Speed-up** | **18.5x** | **45.5x** |

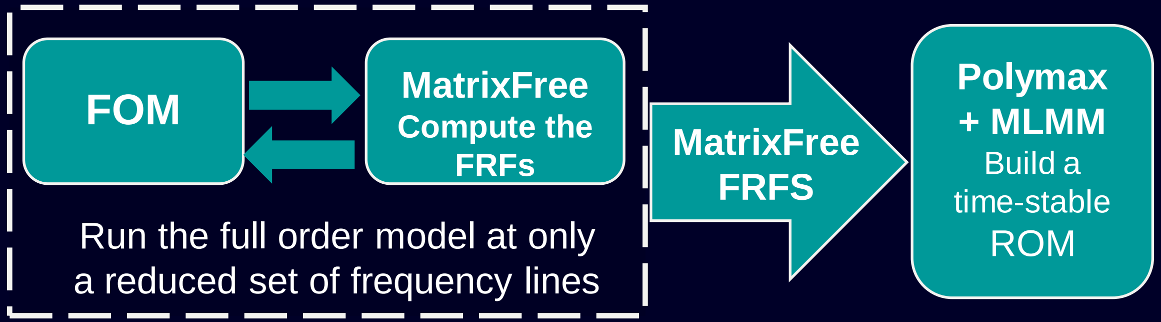

1- Xie, Xiang, Hui Zheng, Stijn Jonckheere, Axel van de Walle, Bert Pluymers, and Wim Desmet. "Adaptive model reduction technique for large-scale dynamical systems with frequency-dependent damping." Computer Methods in Applied Mechanics and Engineering 332 (2018): 363-381.

2- Li, Yue, Onur Atak, Stijn Jonckheere, and Wim Desmet. "Accelerating boundary element methods in wideband frequency sweep analysis by matrix-free model order reduction." Journal of Sound and Vibration 541 (2022): 117323.

# Chaining the MatrixFree Method & Polymax+MLMM

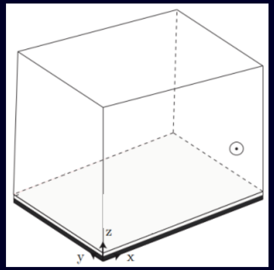
## MatrixFree & Polymax+MLMM

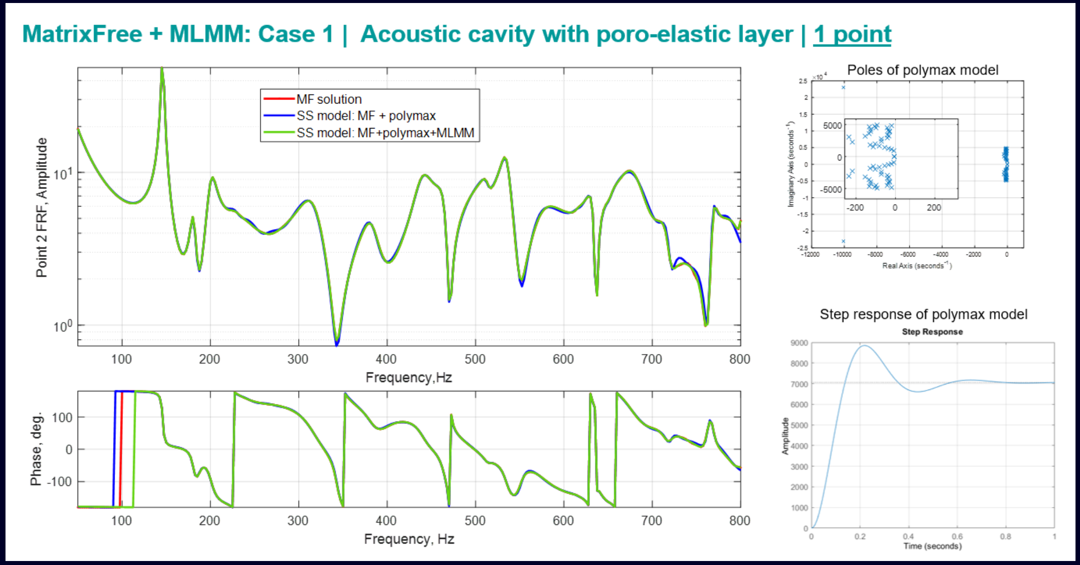
- **Polymax+MLMM** is an FRF-to-ROM method that can generate time stable ROMs from FRF sets
- **MatrixFree** and **Polymax+MLMM** methods are used in a chain to accelerate ROM creation process
- The combined approach provides a **robust way to create time-stable ROMs**, especially **when there is no native time-domain equivalent formulation**




## Acoustic cavity with poro-elastic layer

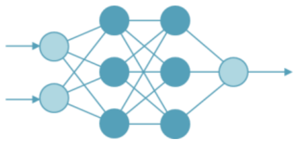
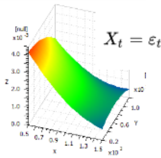
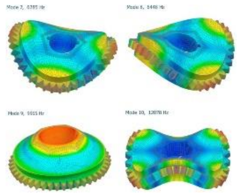
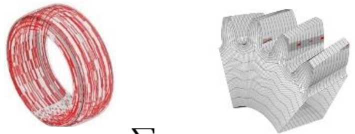
| | | Vigran [24] - top | Carpet [25] - bottom |
|---|---|---|---|
| Bulk density | $\rho_1$ (kg/m$^3$) | 30 | 60 |
| Bulk shear modulus | N (Pa) | $160\cdot10^3 + i30\cdot10^3$ | |
| Bulk Young's modulus | E (Pa) | $430e\cdot10^3 + i100\cdot10^3$ | $20\cdot10^3 + i10\cdot10^3$ |
| Bulk Poisson Ratio | $\nu$ (-) | | 0 |
| Tortuosity | $\alpha_\infty$ (-) | 2.5 | 1 |
| Porosity | $\phi$ (-) | 0.93 | 0.99 |
| Characteristic viscous length | $\Lambda$ (m) | $10\cdot10^{-6}$ | $150\cdot10^{-6}$ |
| Characteristic thermal length | $\Lambda'$ (m) | $100\cdot10^{-6}$ | $220\cdot10^{-6}$ |
| Static flow resistivity | $\sigma$ (kg/m$^3$s) | $80\cdot10^3$ | $20\cdot10^3$ |




1- S. Jonckheere, M. Elkafafy, O. Atak, H. Bériot, K. Janssens, B. Peeters, W. Desmet *"Efficient black box time domain simulation of frequency domain based vibro-acoustic models", Proceedings of ISMA 2022, Leuven, Belgium*

# Overview of ROM Methods

| Data Driven | | | | | Physics Based |
|---|---|---|---|---|---|
| **Category** | **Machine learning** | **Mapping and interpolation models** | **Linear algebra** | **(Petrov)-Galerkin projections** | **Hyper-reduction and PMOR** |
| **Technique** | **Neural networks** | **Response Surface Model Polynomial regression, FRF interpolation…** | **LTI models obtained via linearization** | **Modal CMS, Krylov subspace, multi-scale methods, POD ...** | **Hyper-reduction and PMOR** |
| **Examples** |  *multi-layer perceptrons, convolutional networks…* |  $X_t = \varepsilon_t + \sum_{i=1}^{p} \varphi_i X_{t-i} + \sum_{i=1}^{q} \theta_i \varepsilon_{t-i}$ *RSM, (auto)regressive models, Kriging, Loewner, Polymax* | $\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t)$ $\mathbf{y}(t) = \mathbf{C}\mathbf{x}(t) + \mathbf{D}\mathbf{u}(t)$ *low order linear models (transfer functions, state-space…)* |  *Mode sets, Krylov vectors, physics based ROMs* |  $\boldsymbol{K}(p) = \boldsymbol{K}_0 + \sum_i [\boldsymbol{K}_i^{\lambda}\lambda(E_i,\nu_i) + \boldsymbol{K}_i^{\mu}\mu(E_i,\nu_i)]$ *DEIM, ECSW, PGD, PMOR etc.* |
| **Data source** | 1D 3D CFD TEST | 1D 3D CFD TEST | 1D 3D CFD TEST | 1D 3D CFD TEST | 1D 3D CFD TEST |

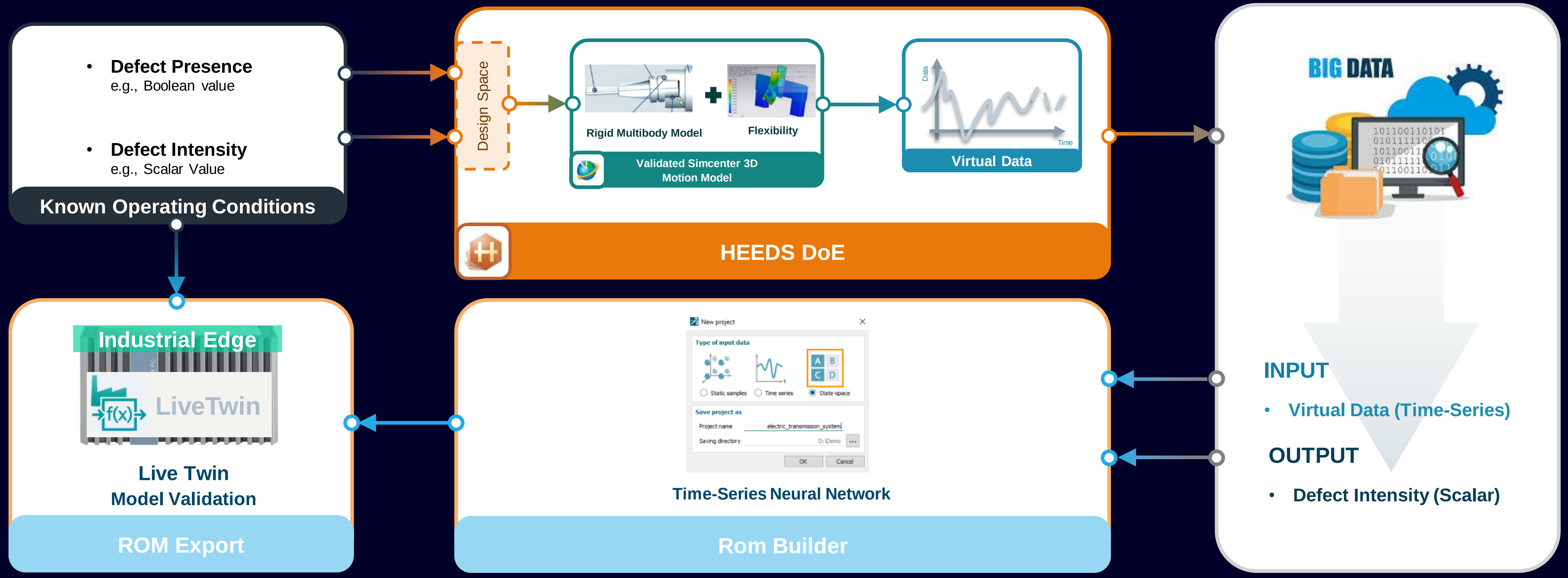
*with* ***LTI****: Linear Time Invariant,* ***POD****: Proper Orthogonal Decomposition,* ***RSM****: Response Surface Model,* ***CMS****: Component Mode Synthesis*
***DEIM****: Discrete Empirical Interpolation Method,* ***ECSW****: Energy Conserving Sampling and Weighting,* ***PGD****: Proper Generalized Decomposition,* ***PMOR****: Parametric Model Oder Reduction*



SIEMENS

# Predicting chip size on a spindle
# Simcenter Motion + HEEDS + ROM Builder

- Simulation models are updated to match measurements with known chip size (20,30,40 micron)
- Missing data is generated via the updated models
- The full data is fed to ROM Builder to generate the ROM





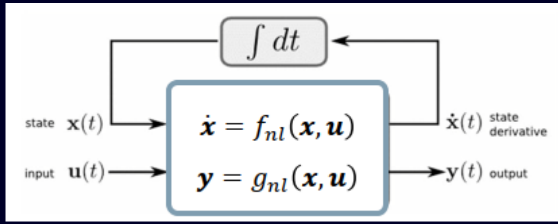

SIEMENS

# Data-Driven Surrogate Modeling via System Identification: NeuralODEs

## What is it?

- **NeuralODEs** (NODE [1] ) is a non-intrusive method to learn a nonlinear **Dynamical Surrogate Model** from data.
- Consists of a **Feed Forward Neural Network** which approximate the *continuos* ODEs: $\dot{\boldsymbol{x}} = f_{NODE}(\boldsymbol{x}, \boldsymbol{u})$
  - It's the generalization of the *discrete* Residual Neural Networks (ResNet).
- Can be extended to system with **external input, noise** and/or **partial knowleadge** of the states varaibles.

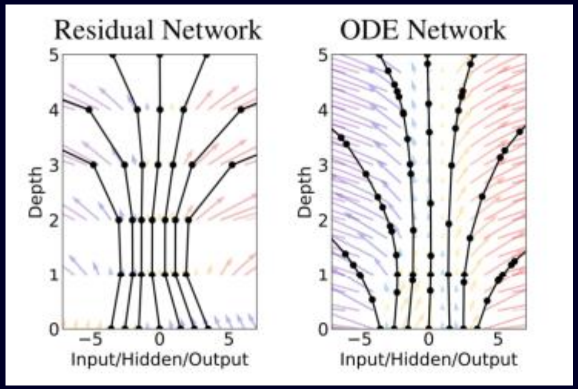
### 1st Order State Space Dynamical System



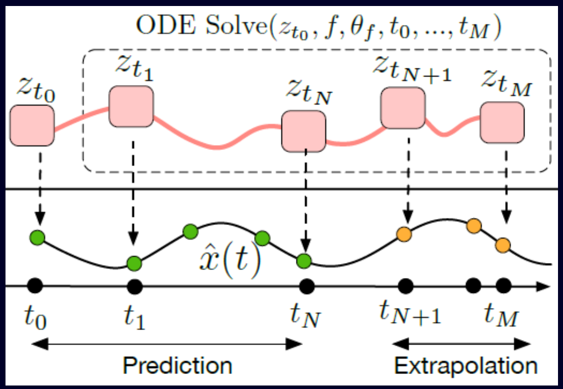

## How?

- **Training**: backpropagation through an ODE Solver.
- **Data-Driven**: It only requires state trajectories .
  - no derivatives, no matrices, no previous knowleadge on ODE or nonlinearities.

### ResNet vs. NeuralODEs




### NODE Prediction and Extrapolation




1 Chen, Ricky TQ, et al. "Neural ordinary differential equations." *Advances in neural information processing systems* 31 (2018).

# Conclusions

- Reduced Order Modelling is a vast and ever-evolving research field
- No single method to fit all!
- The choice of method depends on target application and the corresponding requirements
- Physics-based and Data-driven methods complement each other
- There are various techniques already available in our current product portfolio
- Many more to come:
  - Some already in the roadmaps
  - Some are in research phase

# Contact

**Onur Atak**

Research Engineering Manager – Executable Digital Twins and 3D ROM
DI SW STS SDPRM MECH RTD XDT 3DROM

Siemens Digital Industries Software Limited
Francis House, 112 Hills Road
Cambridge  CB2 1PH, United Kingdom

**E-mail** onur.atak@siemens.com



SIEMENS